

Package ‘cocons’

December 10, 2025

Type Package

Title Covariate-Based Covariance Functions for Nonstationary Spatial Modeling

Version 0.1.5

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Description

Estimation, prediction, and simulation of nonstationary Gaussian process with modular covariate-based covariance functions.

Sources of nonstationarity, such as spatial mean, variance, geometric anisotropy, smoothness, and nugget, can be considered based on spatial characteristics.

An induced compact-supported nonstationary covariance function is provided, enabling fast and memory-efficient computations when handling densely sampled domains.

Encoding UTF-8

LazyData true

License GPL (>= 3)

Depends R (>= 3.5.0)

Imports Rcpp (>= 1.0.10), spam (>= 2.9.1), fields, optimParallel, methods, knitr

LinkingTo Rcpp, BH

BugReports <https://github.com/blasif/cocons/issues>

RoxygenNote 7.3.2

VignetteBuilder knitr

NeedsCompilation yes

Repository CRAN

Date/Publication 2025-12-10 14:10:16 UTC

Contents

cocons-package	3
coco	3
coco-class	6
cocoOptim	6
cocoPredict	8
cocoSim	10
cov_rns	12
cov_rns_classic	12
cov_rns_pred	13
cov_rns_taper	14
cov_rns_taper_pred	14
getAIC	15
getBIC	16
getBoundaries	16
getBoundariesV2	17
getBoundariesV3	18
getBoundariesV4	19
getCIs	19
getCovMatrix	20
getCRPS	21
getDensityFromDelta	22
getDesignMatrix	22
getEstims	23
getHessian	23
getLoglik	24
getLogScore	24
getModelLists	25
getModHess	26
GetNeg2loglikelihood	26
GetNeg2loglikelihoodProfile	27
GetNeg2loglikelihoodREML	28
GetNeg2loglikelihoodTaper	29
GetNeg2loglikelihoodTaperProfile	30
getScale	31
getSpatEffects	31
getSpatMean	32
holes	32
holes_bm	33
is.formula	34
plot,coco,missing-method	34
plotOptimInfo	35
show	36
stripes	36
summary	37
sumsmoothlone	37

 cocons-package

Covariate-based Covariance Functions for Nonstationary Gaussian Processes

Description

Provides routines and methods for estimating and predicting nonstationary Gaussian process models with modular covariate-based covariance functions. Several sources of nonstationarity can be modeled based on spatial information, including a spatial mean, marginal standard deviation, local geometric anisotropy, local nugget, and spatially varying smoothness. Each of these components is modeled separately. An induced compact-supported nonstationary covariance function is provided to speed up computations when handling densely sampled domains. Model parameters are estimated via maximum likelihood (and flavours of it, such as penalized and profile maximum likelihood). A variety of functions are also included to compute prediction metrics and to visualize, simulate, and summarize these types of models. Details of the models can be found in the vignette and in [coco](#).

Disclaimer

This package is provided "as is" without warranty of any kind, either express or implied. Backwards compatibility will not be offered until later versions.

Author(s)

Federico Blasi [aut, cre], <federico.blasi@gmail.com>

Examples

```
## Not run:
  vignette("cocons", package = "cocons")
  methods(class = "coco")

## End(Not run)
```

 coco

Creates a coco S4 object

Description

Creates an S4 object of class `coco`, which is the centerpiece of the **cocons** package. The function provides a set of consistency checks for ensuring the suitability of the different objects involved.

Usage

```
coco(type, data, locs, z, model.list, info, output = list())
```

Arguments

<code>type</code>	(character) One of two available types "dense" or "sparse". See description.
<code>data</code>	(data.frame) A data.frame with covariates information, where <code>colnames(data)</code> matches <code>model.list</code> specification.
<code>locs</code>	(matrix) A matrix with spatial locations.
<code>z</code>	(vector or matrix) A matrix of $n \times r$ response realizations, one realization per column. When considering only one realization, a vector can also be provided.
<code>model.list</code>	(list) A list specifying a model for each source of nonstationarity.
<code>info</code>	(list or NULL) A list specifying characteristics of the coco object.
<code>output</code>	(list or NULL) Empty or the resulting object from running <code>optimParallel</code> , adding to this a list with boundary information (see <code>getBoundaries</code> to check the expected structure).

Details

Two types of coco objects are available, each assuming a different type of covariance matrix for the Gaussian process. Type "dense" builds dense covariance matrices (non zero elements), while type "sparse" builds sparse covariance matrices by tapering the dense covariance matrix with a compact isotropic compact-supported correlation matrix [1]. Type "sparse" allows a set of efficient algorithms, thus making it more suitable for large sample sizes.

An important component of the coco S4 class is the `model.list` specification, involving individual formulas provided as a list, where each of them specifies a covariate-based parametric model for a specific source of nonstationarity. It involves "mean" for the spatial mean, the "std.dev" for the marginal standard deviation, "scale", "aniso" and "tilt", each of them shaping specific aspects of the local spatial geometrically anisotropy structure, "smooth" handling local smoothness, and "nugget" handling the local nugget effect. The models are defined as:

Source	Related to	Description	Model
<i>mean</i>	μ	Spatial mean function	$\mathbf{X}_1\beta$
<i>std.dev</i>	σ^X	Marginal standard deviation	$\exp(0.5\mathbf{X}_2\alpha)$
<i>scale</i>	Σ^X	Local scale	$\exp(\mathbf{X}_3\theta_1)$
<i>aniso</i>	Σ^X	Local geometric anisotropy	$\exp(\mathbf{X}_4\theta_2)$
<i>tilt</i>	Σ^X	(Restricted) local tilt	$\cos(\text{logit}^{-1}(\mathbf{X}_5\theta_3))$
<i>smooth</i>	ν^X	Local smoothness	$(\nu_u - \nu_l)/(1 + \exp(-\mathbf{X}_6\phi)) + \nu_l$
<i>nugget</i>	σ_ϵ^X	Local micro-scale variability	$\exp(\mathbf{X}_7\zeta)$

where β , α , θ_1 , θ_2 , θ_3 , ϕ , and ζ are the parameter vectors of each model, ν_l , and ν_u are the lower and upper bounds limiting the range of variation of the spatially-varying smoothness, and where \mathbf{X}_ℓ relates to the design matrix defined by the specific models for each of the source of nonstationarity.

Lastly, arguments for the "info" list argument involve:

- "lambda.reg": (numeric) a positive scalar specifying the regularization parameter. Larger values discourage highly-smoothed long-tailed covariance functions.
- "lambda.Sigma": (numeric) a positive scalar specifying the penalization parameter for the covariate-driven covariance parameters.

- "lambda.betas": (numeric) a positive scalar specifying the penalization parameter for the covariate-driven spatial mean parameters.
- "sparse.point": (numeric) a cutting point for which smaller coefficients in absolute value will be set to zero after the smoothed L1 penalization optimization. Used in combination with lambda.Sigma and lambda.betas. By default, it is set to 1e-4.
- "smooth.limits": (numeric vector) specifying the range of variation for the spatially varying smoothness (e.g. c(0.5, 2.5)).
- "taper": (numeric) specifying the desired taper function from the spam package (only for "sparse" coco objects).
- "delta": (numeric) specifying the taper range/scale (only for "sparse" coco objects).
- "skip.scale": (integer vector) By default, all covariates are scaled. skip.scale allows to specify the index of those variables in data that should not be scaled during the optimization.

Value

(S4) An S4 object of class coco.

Author(s)

Federico Blasi

References

[1] Furrer, Reinhard, Marc G. Genton, and Douglas Nychka. "Covariance tapering for interpolation of large spatial datasets." *Journal of Computational and Graphical Statistics* 15.3 (2006): 502-523.

See Also

[spam::cov.wend1\(\)](#)

Examples

```
## Not run:
locs <- expand.grid(seq(0,1,length.out = 10),
  seq(0,1,length.out = 10))

toydata <- data.frame('x' = locs[,1])

set.seed(1)
z <- rnorm(100)

model.list <- list('mean' = 0,
  'std.dev' = formula( ~ 1),
  'scale' = formula( ~ 1 + x),
  'aniso' = 0,
  'tilt' = 0,
  'smooth' = 3/2,
  'nugget' = -Inf)
```

```

coco_object <- coco(type = 'dense',
                  data = toydata,
                  locs = as.matrix(locs),
                  z = z,
                  model.list = model.list)

coco_object

## End(Not run)

```

coco-class	<i>An S4 class to store information</i>
------------	---

Description

An S4 class to store information

Slots

`type` (character) One of two available types "dense" or "sparse". See description.

`data` (data.frame) A data.frame with covariates information, where `colnames(data)` matches `model.list` specification

`locs` (numeric matrix) a matrix with `locs` matching `data`

`z` (numeric matrix) A matrix of dimension $n \times p$ with response values

`model.list` (list) A list specifying a model for each aspect of the spatial structure.

`info` (list) a list with information about the `coco` object

`output` (list) if building an already fitted `coco` object (not the standard approach), then requires an output from `Optimparallel` output, including as well boundaries, etc.

Author(s)

Federico Blasi

cocoOptim	<i>Optimizer for coco objects</i>
-----------	-----------------------------------

Description

Estimation the spatial model parameters using the L-BFGS-B optimizer [1].

Usage

```

cocoOptim(coco.object, boundaries = list(), ncores = "auto", safe = TRUE,
          optim.type, optim.control)

```

Arguments

coco.object	(S4) A coco object.
boundaries	(list) If provided, a list containing lower, initial, and upper values for the parameters, as defined by getBoundaries . If not provided, these values are automatically computed via getBoundariesV4 with global lower and upper bounds set to -3 and 3.
ncores	(character or integer) The number of threads to use for the optimization. If set to "auto", the number of threads is chosen based on system capabilities or a fraction of the available cores.
safe	(logical) If TRUE, the function avoids Cholesky decomposition errors due to ill-posed covariance matrices by returning a pre-defined large value. Defaults to TRUE. <ul style="list-style-type: none"> "ml": Classical Maximum Likelihood estimation. "pml": Profile Maximum Likelihood, factoring out the spatial trend for dense objects or the global marginal variance parameter for sparse objects.
optim.type	(character) The optimization approach. Options include:
optim.control	(list) A list of settings to be passed to the optimParallel function [2].

Value

(S4) An optimized S4 object of class `coco`.

Author(s)

Federico Blasi

References

- [1] Byrd, Richard H., et al. "A limited memory algorithm for bound constrained optimization." *SIAM Journal on scientific computing* 16.5 (1995): 1190-1208.
- [2] Gerber, Florian, and Reinhard Furrer. "*optimParallel: An R package providing a parallel version of the L-BFGS-B optimization method.*" *R Journal* 11.1 (2019): 352-358.

See Also

[\[optimParallel\]](#)

Examples

```
## Not run:
model.list <- list('mean' = 0,
                  'std.dev' = formula(~ 1 + cov_x + cov_y),
                  'scale' = formula(~ 1 + cov_x + cov_y),
                  'aniso' = 0,
                  'tilt' = 0,
                  'smooth' = 3/2,
                  'nugget' = -Inf)
```

```

sample_index <- sample(1:dim(holes[[1]])[1],200)

coco_object <- coco(type = 'dense',
                   data = holes[[1]][sample_index, ],
                   locs = as.matrix(holes[[1]][sample_index,1:2]),
                   z = holes[[1]][sample_index, ]$z,
                   model.list = model.list)

optim_coco <- cocoOptim(coco_object,
                       boundaries = getBoundaries(coco_object,-3,3))

plotOptimInfo(optim_coco)

plot(optim_coco)

plot(optim_coco, type = 'ellipse')

plot(optim_coco, type = 'correlations', index = c(2,3,5))

summary(optim_coco,inv.hess = getHessian(optim_coco))

getEstims(optim_coco)

## End(Not run)

```

cocoPredict

Prediction for coco objects

Description

Computes the conditional expectation and standard errors based on the conditional Gaussian distribution for nonstationary spatial models.

Usage

```
cocoPredict(coco.object, newdataset, newlocs, type = 'mean', ...)
```

Arguments

coco.object	(S4) A fitted <code>coco</code> object.
newdataset	(data.frame) A data.frame containing the covariates present in <code>model.list</code> at the prediction locations.
newlocs	(matrix) A matrix specifying the prediction locations, matching <code>newdataset</code> index.
type	(character) Specifies whether to return only the point prediction ('mean') or both the point prediction and prediction standard errors ('pred').

... Additional arguments. If `coco.object` contains multiple realizations, the argument `index.pred` can be used to specify which realization of `coco.object@z` should be used for predictions.

Value

A list containing:

- `systematic`: The systematic component of the conditional expectation.
- `stochastic`: The stochastic component of the conditional expectation.
- `sd.pred`: The standard errors, when `type = 'pred'` is specified.

Author(s)

Federico Blasi

Examples

```
## Not run:

# Stationary model

model.list_stat <- list('mean' = 0,
  'std.dev' = formula(~ 1),
  'scale' = formula(~ 1),
  'aniso' = 0,
  'tilt' = 0,
  'smooth' = 3/2,
  'nugget' = -Inf)

model.list_ns <- list('mean' = 0,
  'std.dev' = formula(~ 1 + cov_x + cov_y),
  'scale' = formula(~ 1 + cov_x + cov_y),
  'aniso' = 0,
  'tilt' = 0,
  'smooth' = 3/2,
  'nugget' = -Inf)

coco_object <- coco(type = 'dense',
  data = holes[[1]][1:100, ],
  locs = as.matrix(holes[[1]][1:100, 1:2]),
  z = holes[[1]][1:100, ]$z,
  model.list = model.list_stat)

optim_coco_stat <- cocoOptim(coco_object,
  boundaries = getBoundaries(coco_object,
  lower.value = -3, 3))

coco_preds_stat <- cocoPredict(optim_coco_stat, newdataset = holes[[2]],
  newlocs = as.matrix(holes[[2]][, 1:2]),
  type = "pred")
```

```

# Update model
coco_object@model.list <- model.list_ns

optim_coco_ns <- cocoOptim(coco_object,
  boundaries = getBoundaries(coco_object,
  lower.value = -3, 3))

coco_preds_ns <- cocoPredict(optim_coco_ns, newdataset = holes[[2]],
  newlocs = as.matrix(holes[[2]][, 1:2]),
  type = "pred")

par(mfrow = c(1, 3))

fields::quilt.plot(main = "full data", holes[[1]][, 1:2],
  holes[[1]]$z, xlim = c(-1, 1), ylim = c(-1, 1))

fields::quilt.plot(main = "stationary se", holes[[2]][, 1:2],
  coco_preds_stat$sd.pred, xlim = c(-1, 1), ylim = c(-1, 1))
fields::quilt.plot(main = "nonstationary se", holes[[2]][, 1:2],
  coco_preds_ns$sd.pred, xlim = c(-1, 1), ylim = c(-1, 1))

## End(Not run)

```

cocoSim

Marginal and conditional simulation of nonstationary Gaussian processes

Description

draw realizations of stationary and nonstationary Gaussian processes with covariate-based covariance functions.

Usage

```
cocoSim(coco.object, pars, n, seed, standardize,
  type = 'classic', sim.type = NULL, cond.info = NULL)
```

Arguments

coco.object	(S4) A <code>coco</code> object.
pars	(numeric vector or NULL) A vector of parameter values associated with <code>model.list</code> . If <code>coco.object</code> is a fitted object, and <code>pars</code> is NULL, it get <code>pars</code> from <code>coco.object@output\$pars</code> (and also sets 'type' to 'diff').
n	(integer) Number of realizations to simulate.
seed	(integer or NULL) Seed for random number generation. Defaults to NULL.

standardize	(logical) Indicates whether the provided covariates should be standardized (TRUE) or not (FALSE). Defaults to TRUE.
type	(character) Specifies whether the parameters follow a classical parameterization ('classic') or a difference parameterization ('diff'). Defaults to 'classic'. For sparse coco objects, only 'diff' is allowed.
sim.type	(character) If set to 'cond', a conditional simulation is performed.
cond.info	(list) A list containing additional information required for conditional simulation.

Details

The argument `sim.type = 'cond'` specifies a conditional simulation, requiring `cond.info` to be provided. `cond.info` is a list including `newdataset`, a `data.frame` containing covariates present in `model.list` at the simulation locations, and `newlocs`, a matrix specifying the locations corresponding to the simulation, with indexing that matches `newdataset`.

The argument `type = 'classic'` assumes a simplified parameterization for the covariance function, with log-parameterizations applied to the parameters `std.dev`, `scale`, and `smooth`.

Value

(matrix) a matrix `dim(data)[1] x n`.

Author(s)

Federico Blasi

See Also

[coco](#)

Examples

```
## Not run:

model.list <- list('mean' = 0,
                  'std.dev' = formula(~ 1 + cov_x + cov_y),
                  'scale' = formula(~ 1 + cov_x + cov_y),
                  'aniso' = 0,
                  'tilt' = 0,
                  'smooth' = 0.5,
                  'nugget' = -Inf)

coco_object <- coco(type = 'dense',
                  data = holes[[1]][1:1000,],
                  locs = as.matrix(holes[[1]][1:1000,1:2]),
                  z = holes[[1]][1:1000,]$z,
                  model.list = model.list)

coco_sim <- cocoSim(coco.object = coco_object,
                  pars = c(0,0.25,0.25, # pars related to std.dev
```

```

        log(0.25),1,-1),      # pars related to scale
        n = 1,
        standardize = TRUE)

fields::quilt.plot(coco_object@locs,coco_sim)

## End(Not run)

```

cov_rns	<i>Dense covariance function (difference parameterization)</i>
---------	--

Description

Dense covariance function (difference parameterization)

Usage

```
cov_rns(theta, locs, x_covariates, smooth_limits)
```

Arguments

theta	vector of parameters
locs	a matrix with locations
x_covariates	design data.frame
smooth_limits	smooth limits

Value

dense covariance matrix

cov_rns_classic	<i>Dense covariance function (classic parameterization)</i>
-----------------	---

Description

Dense covariance function (classic parameterization)

Usage

```
cov_rns_classic(theta, locs, x_covariates)
```

Arguments

theta	vector of parameters
locs	a matrix with locations
x_covariates	design data.frame

Value

dense covariance matrix with classic parameterization

cov_rns_pred	<i>Dense covariance function</i>
--------------	----------------------------------

Description

Dense covariance function

Usage

```
cov_rns_pred(  
  theta,  
  locs,  
  locs_pred,  
  x_covariates,  
  x_covariates_pred,  
  smooth_limits  
)
```

Arguments

theta	vector of parameters
locs	a matrix with locations
locs_pred	a matrix with prediction locations
x_covariates	design data.frame
x_covariates_pred	design data.frame at prediction locations
smooth_limits	smooth limits

Value

dense covariance matrix

cov_rns_taper	<i>Sparse covariance function</i>
---------------	-----------------------------------

Description

Sparse covariance function

Usage

```
cov_rns_taper(
  theta,
  locs,
  x_covariates,
  colindices,
  rowpointers,
  smooth_limits
)
```

Arguments

theta	vector of parameters
locs	a matrix with locations
x_covariates	design data.frame
colindices	from spam object
rowpointers	from spam object
smooth_limits	smooth limits

Value

sparse covariance matrix between locs and pred_locs

cov_rns_taper_pred	<i>Sparse covariance function</i>
--------------------	-----------------------------------

Description

Sparse covariance function

Usage

```
cov_rns_taper_pred(
  theta,
  locs,
  locs_pred,
  x_covariates,
  x_covariates_pred,
  colindices,
  rowpointers,
  smooth_limits
)
```

Arguments

theta	vector of parameters
locs	a matrix with locations
locs_pred	a matrix with prediction locations
x_covariates	design data.frame
x_covariates_pred	design data.frame at prediction locations
colindices	from spam object
rowpointers	from spam object
smooth_limits	smooth limits

Value

sparse covariance matrix at locs

getAIC	<i>Retrieve AIC</i>
--------	---------------------

Description

Retrieve the Akaike information criterion from a fitted coco object.

Usage

```
getAIC(coco.object)
```

Arguments

coco.object	(S4) a fitted coco S4 object.
-------------	-------------------------------

Value

(numeric) the associated AIC value

Author(s)

Federico Blasi

`getBIC`*Retrieve BIC*

Description

Retrieve BIC from a fitted coco object.

Usage`getBIC(coco.object)`**Arguments**`coco.object` (S4) a fitted coco S4 object.**Value**

(numeric) the associated BIC value

Author(s)

Federico Blasi

`getBoundaries`*Simple build of boundaries*

Description

provides a generic set of upper and lower bounds for the L-BFGS-B routine

Usage`getBoundaries(x, lower.value, upper.value)`**Arguments**`x` (S4) or (list) a coco.object or a par.pos list (as output from [getDesignMatrix](#))`lower.value` (numeric vector) if provided, provides a vector filled with values lower.value.`upper.value` (numeric vector) if provided, provides a vector filled with values upper.value.**Value**

(list) a list with boundaries and simple init values for the optim L-BFGS-B routine

Author(s)

Federico Blasi

getBoundariesV2	<i>Simple build of boundaries (v2)</i>
-----------------	--

Description

provides a generic set of upper and lower bounds for the L-BFGS-B routine

Usage

```
getBoundariesV2(coco.object, mean.limits, std.dev.limits,  
scale.limits, aniso.limits, tilt.limits, smooth.limits, nugget.limits)
```

Arguments

coco.object	(S4) a coco object.
mean.limits	(numeric vector) a vector of c(lower,init,upper) values for the associated param.
std.dev.limits	(numeric vector) a vector of c(lower,init,upper) values for the associated param.
scale.limits	(numeric vector) a vector of c(lower,init,upper) values for the associated param.
aniso.limits	(numeric vector) a vector of c(lower,init,upper) values for the associated param.
tilt.limits	(numeric vector) a vector of c(lower,init,upper) values for the associated param.
smooth.limits	(numeric vector) a vector of c(lower,init,upper) values for the associated param.
nugget.limits	(numeric vector) a vector of c(lower,init,upper) values for the associated param.

Value

(list) a list with boundaries for the optim L-BFGS-B routine

Author(s)

Federico Blasi

getBoundariesV3 *Simple build of boundaries (v3)*

Description

provides a generic set of upper and lower bounds for the L-BFGS-B routine

Usage

```
getBoundariesV3(coco.object, mean.limits, global.lower,  
std.dev.max.effects,  
scale.max.effects, aniso.max.effects, tilt.max.effects,  
smooth.max.effects, nugget.max.effects)
```

Arguments

coco.object (S4) a coco object.
mean.limits (numeric vector) a vector of c(lower,init,upper) values for the associated param.
global.lower (numeric vector) a vector of c(lower,init,upper) values for the associated param.
std.dev.max.effects
 (numeric vector) a vector of c(lower,init,upper) values for the associated param.
scale.max.effects
 (numeric vector) a vector of c(lower,init,upper) values for the associated param.
aniso.max.effects
 (numeric vector) a vector of c(lower,init,upper) values for the associated param.
tilt.max.effects
 (numeric vector) a vector of c(lower,init,upper) values for the associated param.
smooth.max.effects
 (numeric vector) a vector of c(lower,init,upper) values for the associated param.
nugget.max.effects
 (numeric vector) a vector of c(lower,init,upper) values for the associated param.

Value

(list) a list with boundaries for the optim L-BFGS-B routine

Author(s)

Federico Blasi

getBoundariesV4 *Simple build of boundaries*

Description

provides a set of upper and lower bounds for the L-BFGS-B routine

Usage

```
getBoundariesV4(coco.object, lower.bound, upper.bound)
```

Arguments

coco.object	(S4) a coco.object.
lower.bound	(numeric vector) lower bound for covariate-driven parameters of the nonstationary covariance function.
upper.bound	(numeric vector) upper bound for covariate-driven parameters of the nonstationary covariance function.

Value

(list) a list with boundaries and simple init values for the optim L-BFGS-B routine

Author(s)

Federico Blasi

getCIs *Compute approximate confidence intervals for a coco object*

Description

Compute approximate confidence intervals for a (fitted) coco object.

Usage

```
getCIs(coco.object, inv.hess, alpha = 0.95)
```

Arguments

coco.object	(S4) a fitted coco S4 object.
inv.hess	(matrix) Inverse of the Hessian. getHessian .
alpha	(numeric) confidence level.

Value

(numeric matrix) a matrix with approximate confidence intervals for each parameter in the model.

Author(s)

Federico Blasi

getCovMatrix

Covariance matrix for "coco" class

Description

Compute the covariance matrix of `coco.object`.

Usage

```
getCovMatrix(coco.object, type = 'global', index = NULL)
```

Arguments

`coco.object` (S4) a fitted `coco()` object.

`type` (character) whether 'global' to retrieve the regular covariance matrix, or 'local' to retrieve global covariance. based on the local aspects of a specific location (not implemented yet).

`index` (integer) index to perform local covariance matrix (not implemented yet).

Value

(matrix or S4) a $n \times n$ covariance matrix (for 'dense' coco objects) or a S4 spam object (for 'sparse' coco objects).

Author(s)

Federico Blasi

Examples

```
## Not run:
model.list <- list('mean' = 0,
                  'std.dev' = formula( ~ 1 + cov_x + cov_y),
                  'scale' = formula( ~ 1 + cov_x + cov_y),
                  'aniso' = 0,
                  'tilt' = 0,
                  'smooth' = 3/2,
                  'nugget' = -Inf)

coco_object <- coco(type = 'dense',
                   data = holes[[1]][1:100,],
```

```
      locs = as.matrix(holes[[1]][1:100,1:2]),
      z = holes[[1]][1:100,]$z,
      model.list = model.list)

optim_coco <- cocoOptim(coco_object,
  boundaries = getBoundaries(coco_object,
    lower.value = -3, 3))

getCovMatrix(optim_coco)

## End(Not run)
```

getCRPS	<i>Based on a set of predictions computes the Continuous Ranked Probability Score</i>
---------	---

Description

Retrieves the Continuous Ranked Probability Score (CRPS) [1].

Usage

```
getCRPS(z.pred, mean.pred, sd.pred)
```

Arguments

z.pred	(numeric vector).
mean.pred	(numeric vector).
sd.pred	(numeric vector).

Value

(numeric vector) retrieves CRPS.

Author(s)

Federico Blasi

References

[1] Gneiting, Tilmann, and Adrian E. Raftery. "Strictly proper scoring rules, prediction, and estimation." *Journal of the American statistical Association* 102.477 (2007): 359-378.

`getDensityFromDelta` *Based on a specific taper scale (delta), retrieves the density of the covariance matrix.*

Description

Based on a specific taper scale (delta), retrieves the density of the covariance matrix.

Usage

```
getDensityFromDelta(coco.object, delta)
```

Arguments

`coco.object` (S4) a fitted `coco()` object.
`delta` (numeric) a delta taper scale (delta).

Value

(numeric vector) the associate density of the tapered covariance matrix.

Author(s)

Federico Blasi

`getDesignMatrix` *Create an efficient design matrix based on a list of aspect models*

Description

Creates a unique design matrix based on model specification for each of the different potentially spatially varying aspects.

Usage

```
getDesignMatrix(model.list, data)
```

Arguments

`model.list` (list) a list of formulas, one for each source of nonstationarity, specifying the models.
`data` (data.frame) a data.frame.

Value

(list) a list with two elements: a design matrix of dimension (n x p), and a `par.pos` object, indexing columns of the design matrix to each of the spatially-varying functions.

Author(s)

Federico Blasi

getEstims	<i>Retrieve estimates from a fitted coco object</i>
-----------	---

Description

Retrieve estimates from a fitted coco object.

Usage

```
getEstims(coco.object)
```

Arguments

coco.object (S4) a fitted coco S4 object.

Value

(list) a list with the estimates parameters for the different aspects

Author(s)

Federico Blasi

getHessian	<i>getHessian</i>
------------	-------------------

Description

numerically approximate the Hessian. Hessians of parameters based on "pml" are based on full likelihoods.

Usage

```
getHessian(coco.object, ncores = parallel::detectCores() - 1,
eps = .Machine$double.eps^(1/4))
```

Arguments

coco.object (S4) a fitted coco object.
ncores (integer) number of cores used for the computation.
eps (numeric) ...

Value

(numeric matrix) a symmetric matrix pxp of the approximated (observed) Hessian

Author(s)

Federico Blasi

getLoglik	<i>Retrieve the loglikelihood value</i>
-----------	---

Description

Retrieve the loglikelihood value from a fitted coco object.

Usage

```
getLoglik(coco.object)
```

Arguments

coco.object (S4) a fitted coco S4 object.

Value

(numeric) wrap for value from a OptimParallel object

Author(s)

Federico Blasi

getLogScore	<i>Based on a set of predictions computes the Log-Score</i>
-------------	---

Description

Computes the Log-Score [1].

Usage

```
getLogScore(z.pred, mean.pred, sd.pred)
```

Arguments

z.pred (numeric vector).
 mean.pred (numeric vector).
 sd.pred (numeric vector).

Value

(numeric vector) retrieves Log-Score.

Author(s)

Federico Blasi

References

[1] Gneiting, Tilmann, and Adrian E. Raftery. "Strictly proper scoring rules, prediction, and estimation." *Journal of the American statistical Association* 102.477 (2007): 359-378.

getModellists

Builds the necessary input for building covariance matrices

Description

Returns a list of parameter vectors for each of the aspects.

Usage

```
getModellists(theta, par.pos, type = 'diff')
```

Arguments

theta	(numeric vector) a vector of length p, where p is the number of parameters for each of the models.
par.pos	(list) a list detailing in which position of each aspect the elements of theta should be placed. Expected to be par.pos output of getDesignMatrix .
type	(character) whether parameters are related to a classical parameterization ('classic') or a difference parameterization 'diff' . Default set to 'diff'.

Value

(list) a list of different spatial aspects and mean required for the cov.rns functions

Author(s)

Federico Blasi

getModHess	<i>Retrieves the modified inverse of the hessian</i>
------------	--

Description

Based on the inverse of the Hessian (based on the difference parameterization for the std.dev and scale parameters), retrieves the modified inverse of the hessian (i.e. std.dev and scale).

Usage

```
getModHess(coco.object, inv.hess)
```

Arguments

coco.object	(S4) a fitted coco S4 object.
inv.hess	(matrix) Inverse of the Hessian.

Value

(numeric matrix) the modified inverse of the hessian matrix

Author(s)

Federico Blasi

GetNeg2loglikelihood	<i>GetNeg2loglikelihood</i>
----------------------	-----------------------------

Description

compute the negative 2 log likelihood based on theta

Usage

```
GetNeg2loglikelihood(theta, par.pos, locs, x_covariates,
smooth.limits, z, n, lambda, safe = TRUE)
```

Arguments

theta	(numeric vector) a vector with parameters values.
par.pos	(list) par.pos list.
locs	(matrix) spatial location matrix.
x_covariates	(data.frame) design matrix.
smooth.limits	(numeric vector) smooth.limits.

z	(numeric vector) a vector of observed values.
n	(integer) dim(z)[1].
lambda	(numeric vector) a vector including lambda.Sigma, lambda.betas, lambda.reg (in this order)
safe	(TRUE/FALSE) if TRUE returns a large pre-defined value under Cholesky errors. Default TRUE.

Value

value

Author(s)

Federico Blasi

GetNeg2loglikelihoodProfile
GetNeg2loglikelihoodProfile

Description

compute the negative 2 log likelihood based on theta

Usage

```
GetNeg2loglikelihoodProfile(theta, par.pos, locs, x_covariates,
smooth.limits, z, n, x_betas, lambda, safe = TRUE)
```

Arguments

theta	(numeric vector) a vector with parameters values.
par.pos	(list) par.pos list.
locs	(matrix) spatial location matrix.
x_covariates	(data.frame) design matrix.
smooth.limits	(numeric vector) smooth.limits.
z	(numeric vector) a vector of observed values.
n	(integer) dim(z)[1].
x_betas	(matrix) or (data.frame) design matrix for the spatial mean.
lambda	(numeric vector) a vector including lambda.Sigma, lambda.betas, lambda.reg (in this order)
safe	(TRUE/FALSE) if TRUE returns a large pre-defined value under Cholesky errors. Default TRUE.

Value

value

Author(s)

Federico Blasi

 GetNeg2loglikelihoodREML

GetNeg2loglikelihoodREML

Description

compute the negative 2 log REML likelihood based on theta

Usage

```
GetNeg2loglikelihoodREML(theta, par.pos, locs, x_covariates, x_betas,
smooth.limits, z, n, lambda, safe = TRUE)
```

Arguments

theta	(numeric vector) a vector with parameters values.
par.pos	(list) par.pos list.
locs	(matrix) spatial location matrix.
x_covariates	(data.frame) design matrix.
x_betas	(matrix) or (data.frame) design matrix for the spatial mean.
smooth.limits	(numeric vector) smooth.limits.
z	(numeric vector) a vector of contrasts.
n	(integer) dim(z)[1].
lambda	(numeric vector) a vector including lambda.Sigma, lambda.betas, lambda.reg (in this order)
safe	(TRUE/FALSE) if TRUE returns a large pre-defined value under Cholesky errors. Default TRUE.

Value

value

Author(s)

Federico Blasi

GetNeg2loglikelihoodTaper
GetNeg2loglikelihoodTaper

Description

compute the negative 2 log likelihood based on theta

Usage

```
GetNeg2loglikelihoodTaper(theta, par.pos, ref_taper, locs,  
x_covariates, smooth.limits, cholS, z, n, lambda, safe = TRUE)
```

Arguments

theta	(numeric vector) a vector with parameters values.
par.pos	(list) par.pos list from getDesignMatrix .
ref_taper	(S4) spam object based on a compact-supported covariance function.
locs	(matrix) spatial location matrix.
x_covariates	(data.frame) design matrix.
smooth.limits	(numeric vector) smooth.limits.
cholS	(S4) Cholesky object from spam.
z	(numeric vector) a vector of observed values.
n	(numeric) dim(z)[1].
lambda	(numeric vector) a vector including lambda.Sigma, lambda.betas, lambda.reg (in this order)
safe	(TRUE/FALSE) if TRUE returns a large pre-defined value under Cholesky errors. Default TRUE.

Value

value

Author(s)

Federico Blasi

GetNeg2loglikelihoodTaperProfile

GetNeg2loglikelihoodTaperProfile

Description

compute the negative 2 log likelihood based on theta

Usage

```
GetNeg2loglikelihoodTaperProfile(theta, par.pos, ref_taper,  
locs, x_covariates, smooth.limits, cholS, z, n, lambda, safe = TRUE)
```

Arguments

theta	(numeric vector) a vector with parameters values.
par.pos	(list) par.pos list.
ref_taper	(S4) spam object based on a taper based covariance function.
locs	(matrix) spatial location matrix.
x_covariates	(data.frame) design matrix.
smooth.limits	(numeric vector) smooth.limits.
cholS	(S4) Cholesky object from spam.
z	(numeric vector) a vector of observed values.
n	(integer) dim(z)[1].
lambda	(numeric vector) a vector including lambda.Sigma, lambda.betas, lambda.reg (in this order)
safe	(TRUE/FALSE) if TRUE returns a large pre-defined value under Cholesky errors. Default TRUE.

Value

(numeric)

Author(s)

Federico Blasi

getScale	<i>Fast and simple standardization for the design matrix.</i>
----------	---

Description

Centers and scale the design matrix.

Usage

```
getScale(x, mean.vector = NULL, sd.vector = NULL)
```

Arguments

x	(S4) or (matrix) a coco object, or a n x p matrix with covariate information to introduce, where the first column is a column of ones.
mean.vector	(numeric vector) if provided, it centers covariates based on this information.
sd.vector	(numeric vector) if provided, it scales covariates based on this information.

Value

(list) a list with a scaled design matrix of dimension n x (p+1), and a set of mean and sd vectors employed to scale the matrix

Author(s)

Federico Blasi

getSpatEffects	<i>Evaluates the spatially-varying functions from a coco object at locs</i>
----------------	---

Description

Evaluates the spatially-varying functions of the nonstationary spatial structure.

Usage

```
getSpatEffects(coco.object)
```

Arguments

coco.object	(S4) a fitted coco S4 object.
-------------	-------------------------------

Value

(list) a list with the different estimated surfaces.

Author(s)

Federico Blasi

getSpatMean	<i>Computes the spatial mean of a (fitted) coco object</i>
-------------	--

Description

Computes the spatial mean of the (fitted) coco object.

Usage

```
getSpatMean(coco.object)
```

Arguments

coco.object (S4) a fitted coco S4 object.

Value

(numeric vector) a vector with the adjusted trend.

Author(s)

Federico Blasi

holes	<i>Holes Data Set</i>
-------	-----------------------

Description

The synthetic "holes" provides a set of training and test data.frame of a Gaussian process realization with a (inherently dense) nonstationary covariance function. Four holes are present in the training dataset, and the task is to predict them.

Usage

```
holes
```

Format

A list with training and test data.frame with rows and variables:

x first spatial coordinate

y second spatial coordinate

cox_x first spatial characteristic

cov_y second spatial characteristic

z response variable

Source

Source of the data

Examples

```
data(holes)
```

holes_bm

Holes with trend + multiple realizations Data Set

Description

The synthetic "holes_bm" provides a set of training and test data.frame of a Gaussian process realization with a (inherently dense) nonstationary covariance function. Four holes are present in the training dataset, and the task is to predict them. This version provides ten independent realizations of the process, as well as considers a spatial mean effect.

Usage

```
holes_bm
```

Format

A list with training, training.z, test, and test.z data.frames with rows and variables:

x first spatial coordinate

y second spatial coordinate

cox_x first spatial characteristic

cov_y second spatial characteristic

cov_z third spatial characteristic

z.i i-th response variable

Source

Source of the data

Examples

```
data(holes_bm)
```

is.formula	<i>check whether an R object is a formula</i>
------------	---

Description

check whether an R object is a formula

Usage

```
is.formula(x)
```

Arguments

x (ANY) an R object.

Value

TRUE/FALSE

Author(s)

Federico Blasi

plot,coco,missing-method	<i>Plot Method for coco objects</i>
--------------------------	-------------------------------------

Description

This method plots objects of class coco.

Usage

```
## S4 method for signature 'coco,missing'
plot(x, y, type = NULL, index = NULL, factr = 0.1, ...)
```

Arguments

x	(S4) A fitted object of class coco.
y	Not used.
type	(character or NULL) The type of plot. NULL or "ellipse" for drawing ellipse of the convolution kernels.
index	(integer vector) For plotting local correlation plots.
factr	(numeric) Factor rate for size of ellipses.
...	Additional arguments passed to quilt.plot .

Value

Several plots are created.

Author(s)

Federico Blasi

`plotOptimInfo` *Plot log info detailed*

Description

plot output of optim

Usage

```
plotOptimInfo(coco.object, ...)
```

Arguments

`coco.object` an optimized `coco.object`
`...` arguments for `par()`

Value

Outputs a sequence of plots detailing parameters during the optimization routine

Author(s)

Federico Blasi

See Also

[cocoOptim\(\)](#)

show	<i>Show Method for Coco Class</i>
------	-----------------------------------

Description

This method show objects of class 'coco'.

Usage

```
## S4 method for signature 'coco'  
show(object)
```

Arguments

object (S4) An object of class 'coco'.

Value

A plot is created.

Author(s)

Federico Blasi

stripes	<i>Stripes Data Set</i>
---------	-------------------------

Description

The synthetic "stripes" provides a set of training and test data.frame of a Gaussian process realization with a (inherently sparse) nonstationary covariance function. Several stripes are present in the training dataset, and the task is to predict them.

Usage

```
stripes
```

Format

A list with training and test data.frame with rows and variables:

x first spatial coordinate

y second spatial coordinate

cox_x first spatial characteristic

cov_y second spatial characteristic

cov_xy third spatial characteristic

z response variable

Source

Source of the data

Examples

```
data(stripes)
```

summary

Summary Method for Coco Class

Description

method summary for objects of class 'coco'.

Usage

```
## S4 method for signature 'coco'
summary(object, inv.hess = NULL)
```

Arguments

object	(S4) An object of class 'coco'.
inv.hess	(numeric matrix or NULL) inverse of the approximated hessian matrix (getHessian)

Value

summary the coco object

Author(s)

Federico Blasi

sumsmoothlone

smoothed-L1 penalization over the covariate-driven parameters

Description

smoothed-L1 penalization over the covariate-driven parameters

Usage

```
sumsmoothlone(x, lambda, alpha = 1e+06)
```

Arguments

x	a vector of parameters to penalize
lambda	penalization hyperparameter
alpha	hyperparameter to approximate L1 penalization. By default 1e6 provides a decent approximation to L1, while also being a differentiable function

Value

penalization

Index

* datasets

- holes, 32
- holes_bm, 33
- stripes, 36

coco, 3, 3, 7, 8, 10, 11

coco(), 20, 22

coco-class, 6

cocons (cocons-package), 3

cocons-package, 3

cocoOptim, 6

cocoOptim(), 35

cocoPredict, 8

cocoSim, 10

cov_rns, 12

cov_rns_classic, 12

cov_rns_pred, 13

cov_rns_taper, 14

cov_rns_taper_pred, 14

getAIC, 15

getBIC, 16

getBoundaries, 4, 7, 16

getBoundariesV2, 17

getBoundariesV3, 18

getBoundariesV4, 19

getCIs, 19

getCovMatrix, 20

getCRPS, 21

getDensityFromDelta, 22

getDesignMatrix, 16, 22, 25, 29

getEstims, 23

getHessian, 19, 23

getLoglik, 24

getLogScore, 24

getModelLists, 25

getModHess, 26

GetNeg2loglikelihood, 26

GetNeg2loglikelihoodProfile, 27

GetNeg2loglikelihoodREML, 28

GetNeg2loglikelihoodTaper, 29

GetNeg2loglikelihoodTaperProfile, 30

getScale, 31

getSpatEffects, 31

getSpatMean, 32

holes, 32

holes_bm, 33

is.formula, 34

optimParallel, 4, 7

plot, coco, missing-method, 34

plot, coco-method

(plot, coco, missing-method), 34

plotOptimInfo, 35

quilt.plot, 34

show, 36

show, coco-method (show), 36

spam::cov.wend1(), 5

stripes, 36

summary, 37

summary, coco-method (summary), 37

sumsmoothlone, 37